Asymmetry of Gamow-Teller β-decay rates in mirror nuclei

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- History of the subject: search for second-class currents
- Motivation to the present work
- Shell model framework
- Results

See also, N.A.S., C.Volpe, Nucl. Phys. A714 (2003) 441

Weak interactions, ECT*, 16-21 June 2003

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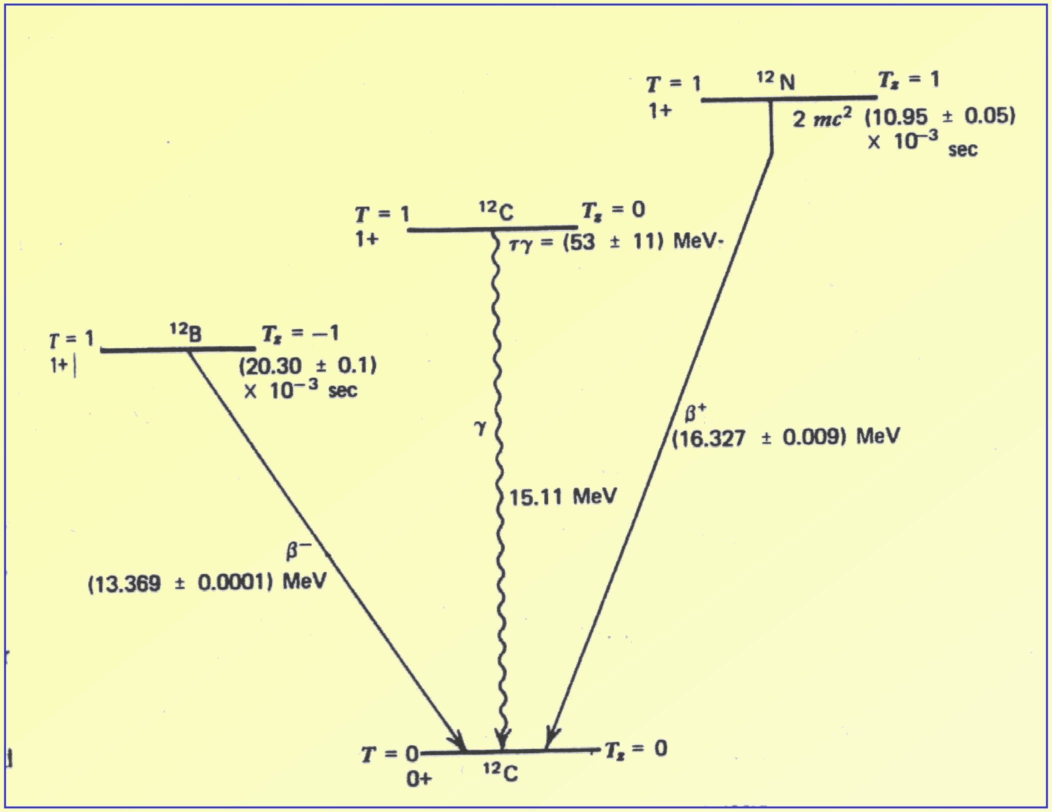
Example of A=12



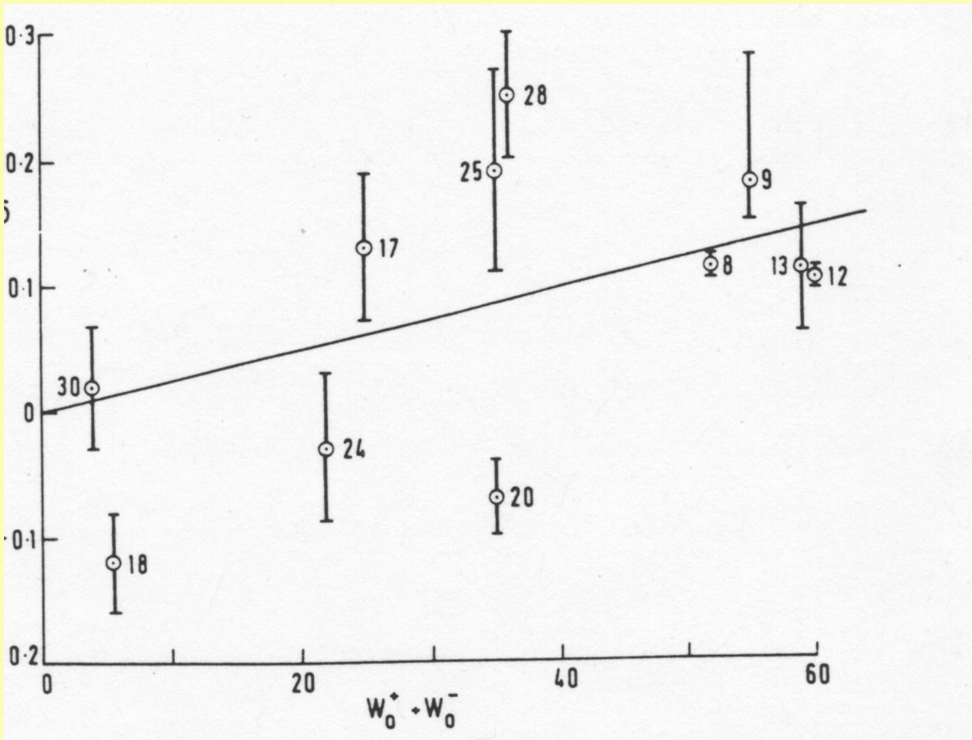
Peterson, Glass (1963); Fisher (1963)

$$\delta = \frac{(ft)_+}{(ft)_-} - 1$$

$$(1^+, T=1) \rightarrow (0^+, T=0): \quad \delta = 13(1)\%$$

$$(1^+, T=1) \rightarrow (2^+, T=0): \quad \delta = 9(2)\%$$

Empirical systematics of asymmetries in allowed Gamow-Teller transitions (earlier 70's)



D.H.Wilkinson (1970)

A < 30 :

11 pairs of mirror nuclei (11 transitions)

$|\Delta T| = 1$; $|\Delta J| = 0, 1$; $\pi_I = \pi_{f.}$

$$\delta = \frac{(ft)_+}{(ft)_-} - 1$$

W_0^+, W_0^- are maximum energy release in the $\beta(+,-)$ decay

Reasons for asymmetry:

1. Weak interaction
2. Nuclear structure effects

Second-class currents in the weak interaction

$$H_{V-A} = \frac{G}{\sqrt{2}} J_\mu^+ j_\mu + H.c. \quad , \qquad J_\mu^+ = V_\mu + A_\mu$$

$$V_\mu = i\overline{\psi}_p \left[g_V \gamma_\mu + \frac{g_M}{2M} \sigma_{\mu\nu} k_\nu + i g_S k_\mu \right] \psi_n$$

vector — weak magnetism — induced scalar

$$A_\mu = i\overline{\psi}_p \left[g_A \gamma_\mu \gamma_5 + \frac{g_T}{2M} \sigma_{\mu\nu} \gamma_5 k_\nu + i g_P k_\mu \gamma_5 \right] \psi_n \qquad k_\mu = (p - p')_\mu$$

axial vector — induced tensor — induced pseudoscalar

Time-reversal invariance: $g_V, g_M, g_S, g_A, g_T, g_P$ are real

$G = C\exp(-i\pi I_2):$

$$GV_\mu^I G^+ = V_\mu^I, \qquad GA_\mu^I G^+ = -A_\mu^I$$
$$GV_\mu^{II} G^+ = -V_\mu^{II}, \qquad GA_\mu^{II} G^+ = A_\mu^{II}$$

S.Weinberg (1958)

CVC: $g_V = 1,\ g_M = \hbar(K_p - K_n)/2Mc,\ g_S = 0$

PCAC: g_A, g_P

Theoretical estimations of second-class currents

$$\left|\frac{g_T}{g_M}\right| = \frac{M_p - M_n}{2M} \approx 7\times 10^{-4}$$

J.F.Donoghue, B.R.Holstein (1982)

$$\left|\frac{g_T}{g_M}\right| = 0.0052 \pm 0.0018$$

QCD sum rules: H.Shiomi (1996)

Experimental searches

nuclear physics

- ft-values

$$\delta = \frac{(ft)_+}{(ft)_-} - 1$$

S.Weinberg (1958) ;J.N.Huffaker, E.Greuling (1963);
K.Kubodera, J.Delorme, M.Rho (1973,1977)

- A=8 system

D.H.Wilkinson, D.E.Alburger (1971)

- (βγ), (βα) correlation experiments

B.R.Holstein (1976,1977,1984)
A=8: L.De Braeckeleer (1992); L.De Braeckeleer et al (1995);
J.F.Amsbaugh, M.Beck, L.De Braeckeleer et al (1997)
A=12: T.Minamisono et al (1998)
A=20: L. Van Elmbdt, J.Deutsch, R.Prieels (1987)

particle physics

$$\tau^- \to \omega\pi^-\nu$$

0.0195±0.0007±0.0011

R.Balest et al (1995)
D.Buskalic et al (1997)

$$\tau^- \to \eta\pi^-\nu$$

$<1.4\times 10^{-4}$

J.Bartelt et al (1996)

$$\bar{\nu}_\mu p \to \mu^+ n$$

L.A.Ahrens et al (1988)

ft-values of mirror transitions

$$\delta = \frac{(ft)_+}{(ft)_-} - 1 = \delta_{scc} + \delta_{nucl}$$

S.Weinberg (1958)

1. **Second-class currents** (scc) contribution

Impulse approximation: $$\delta_{scc} = -\frac{4g_T}{3g_A}\left(W_0^+ + W_0^-\right)$$

J.N.Huffaker, E.Greuling (1963)

W_0^+, W_0^- is the maximum energy release in the β^+, β^- decay

Including off-shell and meson exchange effects:

$$\delta_{scc} = -4\frac{\lambda}{g_A}J + \frac{4}{3g_A}\left(\frac{1}{2}\lambda L - \zeta\right)\left(W_0^+ + W_0^-\right), \quad \zeta = g_T + g'_T$$

K.Kubodera,J.Delorme,M.Rho (1973)

(<2-5%)

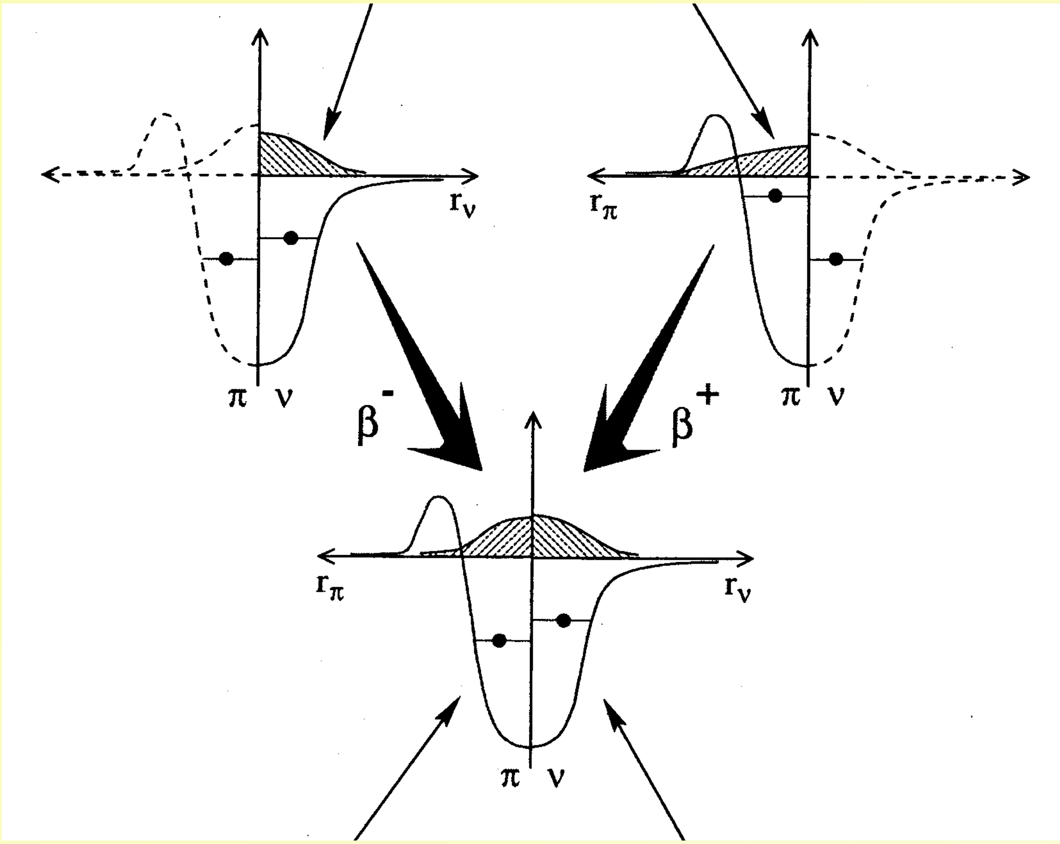
2. **Nuclear structure effects**

- **Coulomb and isospin non-conserving nuclear forces** **(~6-18%)**
- second-forbidden matrix elements (< -1.5%)
- induced weak currents (weak magnetism, induced pseudoscalar) (~ ± 0.5%)
- screening corrections (~ -0.02%)
- radiative corrections (~ ±0.04 %)

R.J.Blin-Stoyle, M.Rosina (1965); D.H.Wilkinson (1971); J.Blomqvist (1971); I.S.Towner (1973)

Binding energy effect

Radial dependence of the nucleon wave function

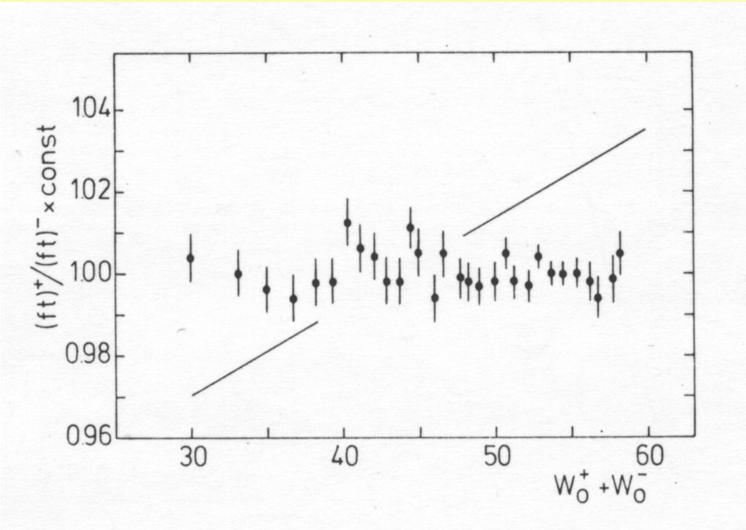


Coulomb and nuclear potential

$$u(r) \propto \exp\left(-\sqrt{\frac{2M\,(E_0 - E)}{\hbar^2}}\;\; r \right)$$

Figure from J.C.Thomas, Ph.D.Thesis (2002)

A=8 system



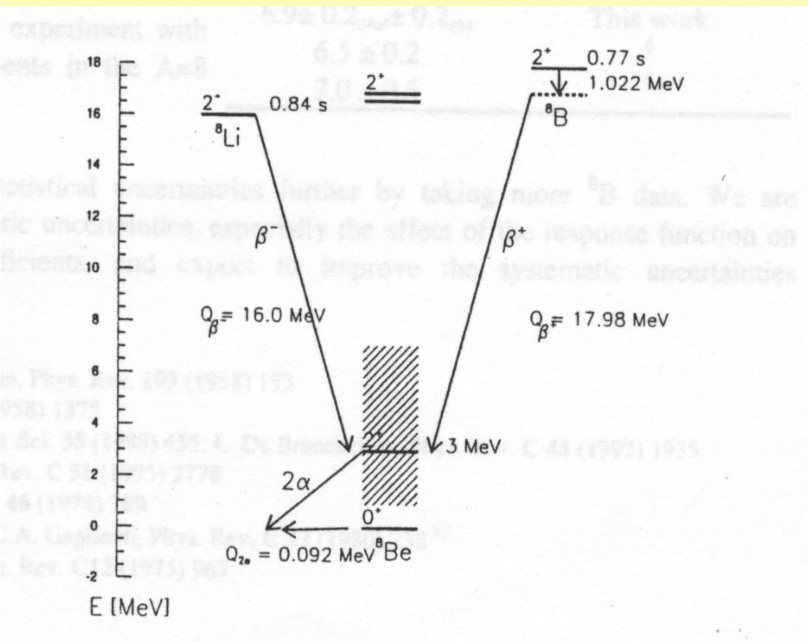
$$\delta_{scc} = -\frac{4 g_T}{3 g_A}\left(W_0^+ + W_0^-\right)$$

$$\frac{N^-(E_x)/f(W_0^-)}{N^+(E_x)/f(W_0^+)} = const \times \left[1 + \xi\left(W_0^+ + W_0^-\right)\right]$$

$$|g_T| < 7\times 10^{-4}$$

D.H.Wilkinson, D.E.Alburger (1971)

Correlation experiments



A=8: (βα)-correlation

g_M, g_T

L. De Braeckeleer (1992); L. De Braeckeleer et al (1995);
J.F.Amsbaugh, M.Beck, L. De Braeckeleer et al (1997)

A=12: β-ray angular distribution from aligned ^{12}B and ^{12}N

$$2M\, g_T / g_A = +0.22 \pm 0.05(stat) \pm 0.15(syst) \pm 0.05$$

T.Minamisono et al (1998)

From: J.F.Amsbaugh, et al (1997)

Motivation

- There are still searches for the existence of the second-class currents

  β-spectroscopy in A=21,25 (Experiment E398 in GANIL, 2003)   B.Blank, J.C.Thomas

- New data on GT transition rates for p and sd-shell nuclei (A=8,9,12,13,17,20,21,25,28,31,33,35)

- Advances in the theoretical description of p- and sd-shell nuclei compared to early 70's

  USD interaction (Wildenthal, 1984);
  Isospin non-conserving Hamiltonians (E.Ormand, B.A.Brown, 1989)

The aim of the study: Systematical calculation of major nuclear structure contributions to asymmetries of ft-values for mirror transition in p- and sd-shell nuclei within the nuclear shell model

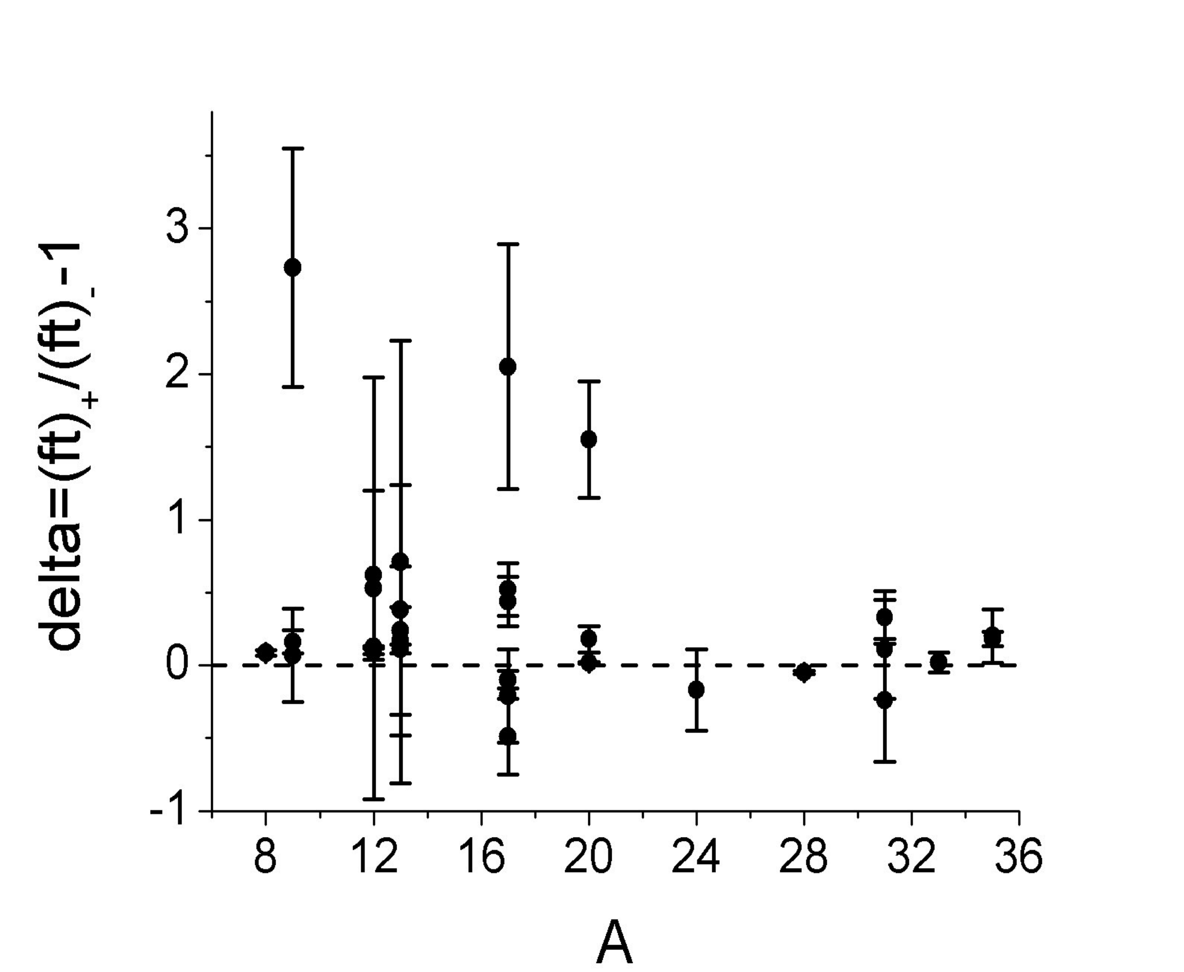
Difficulties:

- Accuracy of the nuclear wave functions
- PCAC: renormalization of g_A (g_A^{eff} =0.77 g_A)

Empirical systematics of asymmetries in allowed Gamow-Teller transitions

$$\delta = \frac{(ft)_+}{(ft)_-} - 1$$

A < 40 :
12 pairs of mirror nuclei (30 transitions)
$|\Delta T| = 1$; $|\Delta J| = 0, 1$; $\pi_I = \pi_f$.



J.C.Thomas, Ph.D. thesis (2002)

Shell Model formalism

$$\delta = \frac{(ft)_+}{(ft)_-} - 1 = \left|\frac{M_-}{M_+}\right|^2 - 1$$

$$M_{\pm} \equiv \left\langle f \left\| \sum_k \sigma(k)\tau_{\pm}(k) \right\| i \right\rangle = \sum_{j_1, j_2, \pi} X(j_1, j_2, J_i, J_f, J_\pi) \underbrace{S^{1/2}(j_2, J_f, J_\pi) S^{1/2}(j_1, J_i, J_\pi)}\Omega^{\pi}_{j_1 j_2}$$

Spectroscopic factors from isospin non-conserving shell model Hamiltonian

Ormand,Brown (1989)

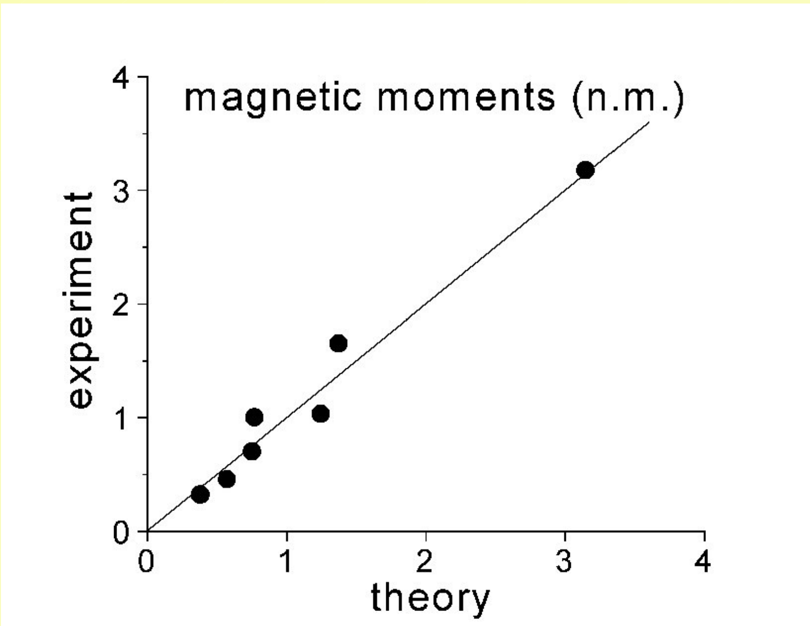
$$\Omega^{\pi}_{j_1 j_2} = \int \underbrace{R^{\pi}_{n_1 j_1 l_1}(r) R^{\pi}_{n_2 j_2 l_2}(r)} r^2 dr$$

Radial wave functions are obtained from Woods-Saxon potential with Coulomb and charge-dependent corrections

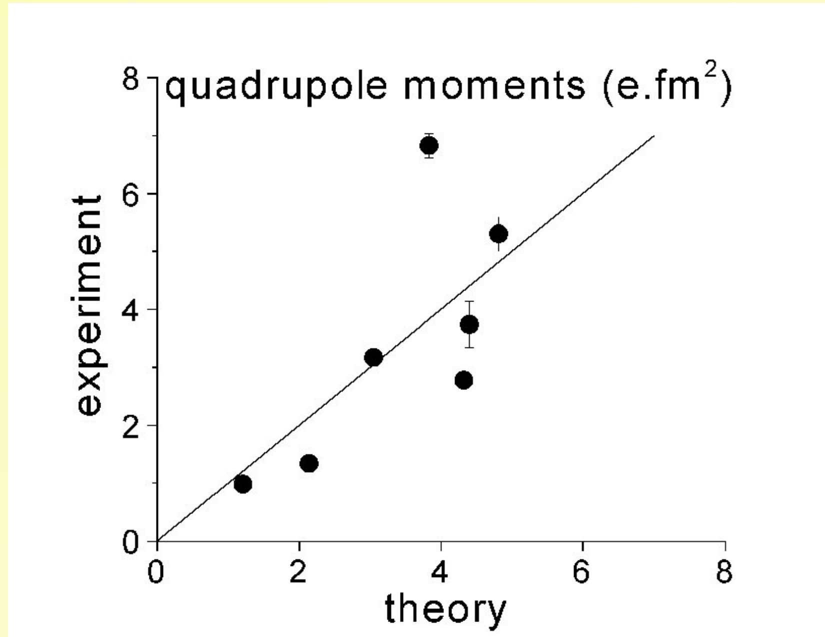
$$V(r) = -V_{ws} f(r) - V_{ls} \frac{r_0^2}{r} \frac{d}{dr} (f(r)) \vec{l} \cdot \vec{s} + V_c h(r) + V_{sym},$$

$$f(r) = \frac{1}{1 + \exp((r - R_0)/a)}, h(r) = \begin{cases} \frac{1}{r} \\ \frac{1}{2R_0}\left(3 - \frac{r^2}{R_0^2}\right) \end{cases}$$

Test of the wave functions: static electromagnetic moments for p-shell nuclei

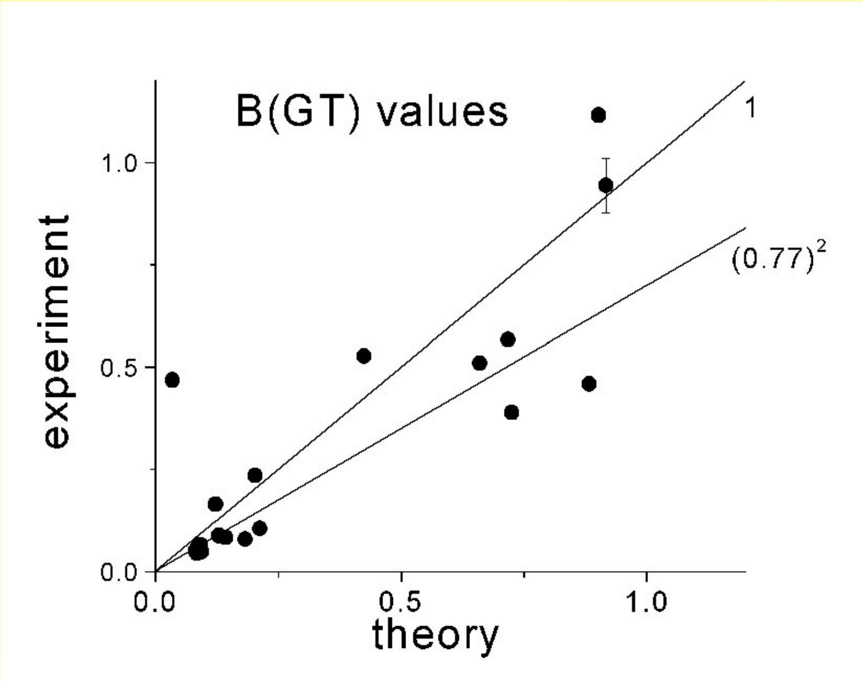


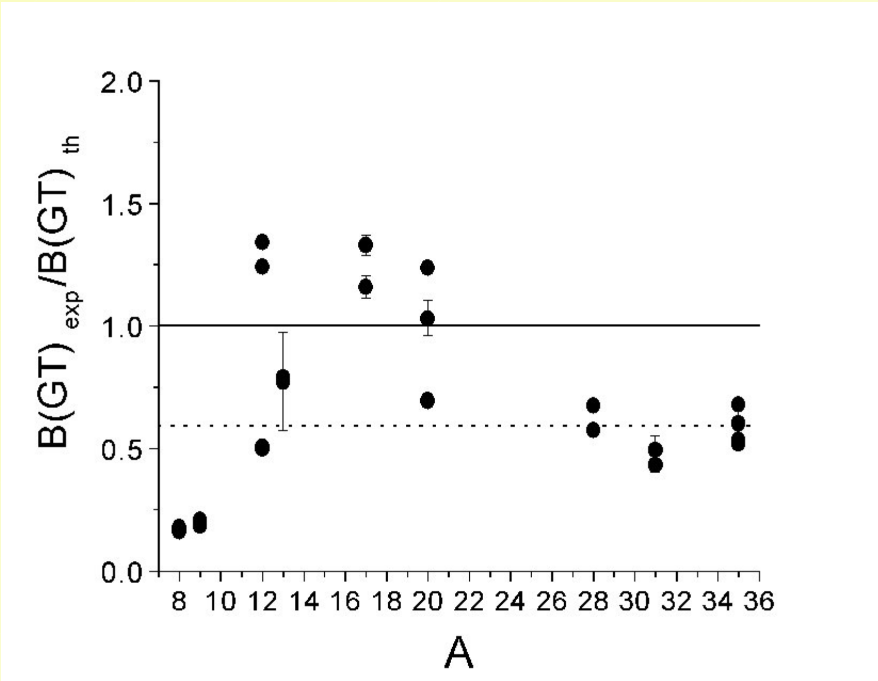
$$g_\pi^s = 5.58\,,\; g_\pi^l = 1.0$$
$$g_\nu^s = -3.82\,,\; g_\nu^l = 0.0$$



$$e_\pi = 1.35\,,\; e_\nu = 0.35$$

B(GT)-values for mirror transitions in p- and sd-shell nuclei





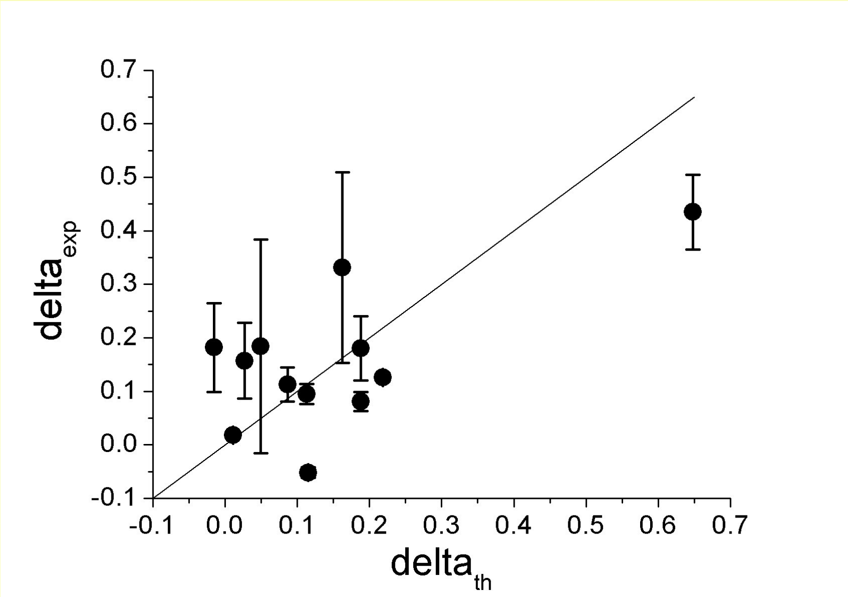
B(GT)-values for mirror transitions in p-shell nuclei

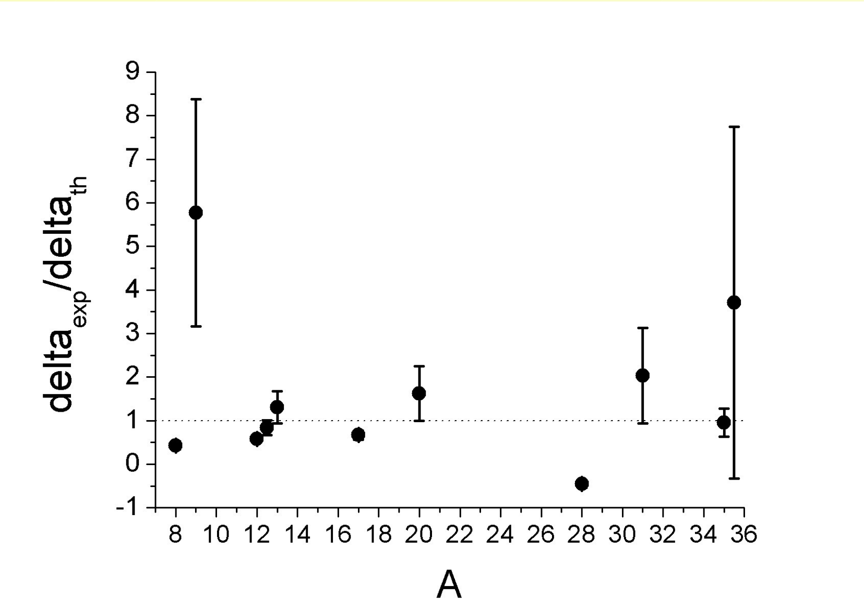
	$J_i^\pi;T_i$	$J_f^\pi;T_f$	INC + HO	IC + WS	INC + WS	(INC + WS)*	EXP
$^8Li(\beta^-)^8Be$	$2^+;1$	$2_1^+;0$	0.1170	0.0265	0.1006	0.0984	0.0158(2)
$^8B(\beta^+)^8Be$	$2^+;1$	$2_1^+;0$	0.1135	0.0082	0.0863	0.0828	0.0146(2)
$^9Li(\beta^-)^9Be$	$\frac{3}{2}^-;\frac{3}{2}$	$\frac{3}{2}{}_1^-;\frac{1}{2}$ (g.s.)	0.1498	0.1052	0.1375	0.1417	0.0296(18)
$^9C(\beta^+)^9B$	$\frac{3}{2}^-;\frac{3}{2}$	$\frac{3}{2}{}_1^-;\frac{1}{2}$ (g.s.)	0.1517	0.0740	0.1310	0.1379	0.0256(3)
$^{12}B(\beta^-)^{12}C$	$1^+;1$	$0_1^+;0$ (g.s.)	0.5078	0.4250	0.4703	0.4250	0.5272(18)
		$2_1^+;0$	0.1038	0.0979	0.0990	0.0954	0.0476(2)
$^{12}N(\beta^+)^{12}C$	$1^+;1$	$0_1^+;0$ (g.s.)	0.5045	0.3629	0.4272	0.3488	0.4682(31)
		$2_1^+;0$	0.1045	0.0875	0.0917	0.0857	0.0435(8)
$^{13}B(\beta^-)^{13}C$	$\frac{3}{2}^-;\frac{3}{2}$	$\frac{1}{2}{}_1^-;\frac{1}{2}$ (g.s.)	0.767	0.6861	0.7206	0.7181	0.5671(72)
$^{13}O(\beta^+)^{13}N$	$\frac{3}{2}^-;\frac{3}{2}$	$\frac{1}{2}{}_1^-;\frac{1}{2}$ (g.s.)	0.752	0.6095	0.6580	0.6609	0.5097(130)

B(GT)-values for mirror transitions in sd-shell nuclei

	$J_i^\pi;T_i$	$J_f^\pi;T_f$	INC + HO	IC + WS	INC + WS	(INC + WS)*	EXP
$^{17}N(\beta^-)^{17}O$	$\frac{1}{2}^-;\frac{3}{2}$	$\frac{3}{2}^-_1;\frac{1}{2}$	0.2079	0.2333	0.2021	0.2023	0.2342(89)
$^{17}Ne(\beta^+)^{17}F$	$\frac{1}{2}^-;\frac{3}{2}$	$\frac{3}{2}^-_1;\frac{1}{2}$	0.1356	0.2216	0.1272	0.1228	0.1632(50)
$^{20}F(\beta^-)^{20}Ne$	$2^+;1$	$2^+_1;0$	0.0921	0.0948	0.0922	0.0924	0.0645(1)
$^{20}Na(\beta^+)^{20}Ne$	$2^+;1$	$2^+_1;0$	0.0935	0.1098	0.1085	0.0914	0.0633(4)
$^{20}O(\beta^-)^{20}F$	$0^+;2$	$1^+_1;1$	0.9124	0.9571	0.9183	0.9031	1.1166(48)
$^{20}Mg(\beta^+)^{20}Na$	$0^+;2$	$1^+_1;1$	0.9120	0.9439	0.9054	0.9171	0.9445(666)
$^{28}Al(\beta^-)^{28}Si$	$3^+;1$	$2^+_1;0$	0.1458	0.1444	0.1435		0.0825(1)
$^{28}P(\beta^+)^{28}Si$	$3^+;1$	$2^+_1;0$	0.1449	0.1303	0.1287		0.0870(9)
$^{31}Al(\beta^-)^{31}Si$	$\frac{5}{2}^+;\frac{5}{2}$	$\frac{3}{2}^+_1;\frac{3}{2}$ (g.s.)	0.2120	0.2090	0.2124		0.1049(122)
$^{31}Ar(\beta^+)^{31}Cl$	$\frac{5}{2}^+;\frac{5}{2}$	$\frac{3}{2}^+_1;\frac{3}{2}$ (g.s.)	0.1947	0.1958	0.1827		0.0788(53)
$^{35}S(\beta^-)^{35}Cl$	$\frac{3}{2}^+;\frac{3}{2}$	$\frac{3}{2}^+_1;\frac{1}{2}$ (g.s.)	0.0861	0.0870	0.0865		0.0588(2)
$^{35}K(\beta^+)^{35}Ar$	$\frac{3}{2}^+;\frac{3}{2}$	$\frac{3}{2}^+_1;\frac{1}{2}$ (g.s.)	0.0846	0.0843	0.0824		0.0496(84)
$^{35}P(\beta^-)^{35}S$	$\frac{1}{2}^+;\frac{5}{2}$	$\frac{1}{2}^+_1;\frac{3}{2}$	0.9052	0.8572	0.8836	0.8861	0.4591(70)
$^{35}Ca(\beta^+)^{35}K$	$\frac{1}{2}^+;\frac{5}{2}$	$\frac{1}{2}^+_1;\frac{3}{2}$	0.8572	0.7435	0.7257	0.7458	0.3891(154)

Asymmetry of mirror transitions





Asymmetry of mirror transitions

A	$J_i^\pi;T_i$	$J_f^\pi;T_f$	INC + HO	IC + WS	INC + WS	(INC + WS)*	EXP
8	$2^+;1$	$2_1^+;0$	3.10	13.21	16.65	18.82	8.4 ± 1.8
9	$\frac{3}{2}^-;\frac{3}{2}$	$\frac{3}{2}_1^-;\frac{1}{2}$ (g.s.)	−1.26	6.4	4.95	2.72	16 ± 8
12	$1^+;1$	$0_1^+;0$ (g.s.)	0.70	9.4	10.09	21.85	12.6 ± 0.8
		$2_1^+;0$	−0.65	8.45	7.92	11.33	9.5 ± 1.9
13	$\frac{3}{2}^-;\frac{3}{2}$	$\frac{1}{2}_1^-;\frac{1}{2}$ (g.s.)	2.10	7.26	9.50	8.65	11.3 ± 3.2
17	$\frac{1}{2}^-;\frac{3}{2}$	$\frac{3}{2}_1^-;\frac{1}{2}$	53.26	5.29	58.90	64.80	44 ± 7
20	$0^+;2$	$1_1^+;1$	0.04	1.41	1.42	−1.53	18 ± 8
20(a)	$2^+;1$	$2_1^+;0$	−1.45	−13.64	−14.99	1.11	1.8 ± 0.7
21	$\frac{5}{2}^+;\frac{3}{2}$	$\frac{3}{2}_1^+;\frac{1}{2}$ (g.s.)	−3.96	3.49	−0.56		
		$\frac{5}{2}_1^+;\frac{1}{2}$	−1.71	2.35	0.60		
		$\frac{7}{2}_1^+;\frac{1}{2}$	2.78	1.44	4.23		
25	$\frac{5}{2}^+;\frac{3}{2}$	$\frac{5}{2}_1^+;\frac{1}{2}$ (g.s.)	−1.09	2.24	1.11		
		$\frac{3}{2}_1^+;\frac{1}{2}$	9.21	2.83	12.39		
		$\frac{7}{2}_1^+;\frac{1}{2}$	7.76	3.83	11.23		
		$\frac{5}{2}_2^+;\frac{1}{2}$	−3.17	−2.52	−5.58		
28	$3^+;1$	$2_1^+;0$	0.61	10.85	11.54		-5 ± 1
31	$\frac{5}{2}^+;\frac{5}{2}$	$\frac{3}{2}_1^+;\frac{3}{2}$ (g.s.)	8.88	6.72	16.27		33 ± 18
35	$\frac{3}{2}^+;\frac{3}{2}$	$\frac{3}{2}_1^+;\frac{1}{2}$ (g.s.)	1.82	3.13	4.96		18.4 ± 20.0
35	$\frac{1}{2}^+;\frac{5}{2}$	$\frac{1}{2}_1^+;\frac{3}{2}$	5.59	15.28	21.75	18.81	18 ± 5

Conclusions and perspectives

A way to continue would be:

- Substitute WS wave functions by the Hartree-Fock wave functions
- Calculate corrections (electromagnetic, second-forbidden, …)
- Evaluate J,L and extract limits on (ξ,λ) of the KDR model

However, it was not done due to following reasons:

- Reproduction of the data (μ,Q) is within 10 –20%
- B(GT) values and asymmetry δ depend on the choice of the Hamiltonian and potential parameters
- Model restrictions (light nuclei, vicinity of the continuum,…)

Improvement in the theoretical description of nuclear states is required !

Correlation experiments could be more suitable in searches for g_{IT}