Doubly Charged Higgs, Neutrino Masses and The LHC

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Triumf

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**Motivation**

- Orthodox view of neutrino mass generation is the *seesaw mechanism*

\[
m_{\nu} \sim \frac{y}{M_N} L L \phi \phi \xrightarrow{SSB} \frac{y v^2}{M_N} \nu_L \nu_L
\]

where \( L, \phi \) are the SM the lepton and Higgs doublets. \( v = 246 \text{ GeV} \).

- Require mass of right-handed singlet neutrino \( N_R \) to be \( M_N > 10^{12} \text{ GeV} \)

- Elegantly connects to GUT’s. *Very difficult to test experimentally*

- Attempts to lower the scale to \( \text{TeV} \) requires fine tuning

- Are SM singlet fermions necessary for neutrino mass generation.
Alternative to Seesaw

• Without $N_R$ how do neutrinos get masses?
  1. Extend the Higgs sector
     (a) Group theory demands that the new fields be Higgs singlet and/or triplet
     (b) $m_\nu$ radiatively induced.
     (c) All new physics is at the the TeV scale.
  2. R-parity violating supersymmetry $LLE$ and $LQE$ terms
Ingredients

- The gauge group is SM $SU(2) \times U(1)$
- Add to the SM the set of Higgs fields. Matter fields remain minimal
  1. One triplet $T$ with SM q.n $(1, 2)$ carries no lepton number

\[
T = \begin{pmatrix} T^0 & \frac{T^-}{\sqrt{2}} \\ \frac{T^-}{\sqrt{2}} & T^{--} \end{pmatrix}_{-2}
\]

- A doubly charged singlet Higgs $\Psi^{++}$ with q.n. $(0, 4)$ with lepton number $2$
- New Yukawa term $Y_{ab} \bar{e}_a^c e_{bR}^c \Psi$ is allowed. $a, b$ are family indices.
- No $LLT$ term.
- The rest of the Yukawa terms are same as SM
The Scalar Potential

- Lepton number violation is introduced in the Higgs potential

\[
V(\phi, T, \psi) = -\mu^2 \phi^\dagger \phi + \lambda_\phi (\phi^\dagger \phi)^2 - \mu_T^2 Tr(T^\dagger T) + \lambda_T [Tr(T^\dagger T)]^2 + \lambda_T' Tr(T^\dagger T T^\dagger T)
\]

\[
+ m^2 \Psi^\dagger \Psi + \lambda_\Psi (\Psi^\dagger \Psi)^2 + \kappa_1 Tr(\phi^\dagger \phi T^\dagger T) + \kappa_2 \phi^\dagger T T^\dagger \phi + \kappa_\Psi \phi^\dagger \phi \Psi^\dagger \Psi
\]

\[
+ \rho Tr(T^\dagger T \Psi^\dagger \Psi) + \left[ \lambda(\tilde{\phi}^T T \tilde{\phi} \Psi) - M(\phi^T T^\dagger \phi) + h.c. \right].
\]

- It is natural in the technical sense \( \lambda, M \rightarrow 0 \) lepton symmetry is restored.
- Majorana neutrino mass must involve \( \lambda \)
- \( \mu^2_T \) is positive so that there is SSB for the triplet field.
- Counting physical spin zero field
  1. Neutral scalars \( h^0, t^0 \) from mixing of \( \phi^0, T^0 \).
  2. Pseudoscalar \( t_\alpha \) from the imaginary part of \( T^0 \)
  3. A pair of charge scalar \( P^\pm \) originates from \( T^\pm \)
  4. Two pairs of doubly charged scalars \( P_{1,2}^{\pm \pm} \) from the mixing of \( T^{\pm \pm}, \Psi^{\pm \pm} \)
Mass Range for Scalars

- $v_T$ is constrained by $\rho = 1.002^{+0.0007}_{-0.0009}$ parameter

$$M_W^2 = \frac{g^2}{4}(v^2 + 2v_T^2), \quad M_Z^2 = \frac{g^2}{4\cos^2 \theta_W}(v^2 + 4v_T^2),$$

We get $v_T < 4\text{GeV}$.

- Minimizing $V$ yields

$$-\mu^2 + \lambda_\phi v^2 + \frac{1}{2}\kappa_+ v_T^2 - \sqrt{2}Mv_T = 0,$$

$$-\mu_T^2 + \lambda_+ v_T^2 + \frac{1}{2}\kappa_+ v^2 - \frac{v^2}{\sqrt{2}} \left( \frac{M}{v_T} \right) = 0,$$

where $\kappa_+ = \kappa_1 + \kappa_2$ and $\lambda_+ = \lambda_T + \lambda'_T$.

- The limiting cases are
  1. $M \sim v_T$ conditions satisfied by $\mu_T \sim v_T$. Parameters not fine tuned.
  2. $M \sim v_T$ needs $\mu_T^2 \sim v^3/v_T$. Appears not natural
**Scalar Mass contd**

- Keeping all the parameters perturbative $< 4\pi$. Singly charged Higgs between 200 and 600 GeV
- The pair of doubly charged Higgs from a two level system. They mix i.e. mass and weak eigenstates are different

$$
\begin{pmatrix}
P_1^{\pm \pm} \\
P_2^{\pm \pm}
\end{pmatrix} =
\begin{pmatrix}
cos \delta & \sin \delta \\
-\sin \delta & \cos \delta
\end{pmatrix}
\begin{pmatrix}
T^{\pm \pm} \\
\Psi^{\pm \pm}
\end{pmatrix}
$$

- Mixing angle is given by

$$
\sin 2\delta = \left[ 1 + \left( \frac{2m^2 + (2\lambda_T^\prime + \rho)v^2_T}{2\lambda v^2} + \frac{\kappa_2 + \kappa_\Psi}{2\lambda} - \frac{\omega}{\lambda} \right)^2 \right]^{-\frac{1}{2}}
$$

- For Case A, if $m^2 \lesssim v^2$, the mixing can be large and close to maximal. But if $m^2 \gg v^2$, the mixing will be small, which is expected since the two states are widely split
- For case B, large mixing can be achieved only if a cancellation occurs between the various parameter
More Masses

• For the masses
  1. $M \sim v_T$: The mass of $P_{1,2}^{\pm\pm}$ is expected to be in the range $200 - 600 \text{ GeV}$ if $m \lesssim 1 \text{ TeV}$. Otherwise $P_{2}^{\pm\pm} > \text{TeV}$.
  2. $M \sim v \gg v_T$: Here, only the mass of $P_{1}^{\pm\pm}$ is expected to be at the weak scale. All others $\gg \text{TeV}$. Out of reach of LHC
Radiative Neutrino Masses

Active neutrinos have masses from 2-loop effect

\[(a)\]

\[
W^- \xrightarrow{P_{1,2}^+} \nu_{aL} \rightarrow \nu_{bL}^c \rightarrow l^c_b \rightarrow W^- \]

\[(b)\]

\[
W^- \xrightarrow{T^{++}} \nu_{aL} \rightarrow \nu_{bL}^c \rightarrow L^c_b \rightarrow W^- \]

The neutrino mass matrix is

\[
(m_\nu)_{ab} = \frac{1}{\sqrt{2}} g^4 m_a m_b v_T Y_{ab} \sin(2\delta) \left[ I(M_W^2, M_{P_1}^2, m_a, m_b) - I(M_W^2, M_{P_2}^2, m_a, m_b) \right],
\]

where \(a, b = e, \mu, \tau\). The integral \(I\) is

\[
I(M_W^2, M_{P_i}^2, m_a^2, m_b^2) = \int \frac{d^4q}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m_a^2} \frac{1}{k^2 - M_W^2} \frac{1}{q^2 - M_W^2} \frac{1}{q^2 - m_b^2} \frac{1}{(k - q)^2}.
\]
Neutrino Mass matrix

- In the limit $M_{P_1,2} > M_W$

$$I(M_W^2, M_{P_i}^2, 0, 0) \sim \frac{1}{(4\pi)^4} \frac{1}{M_{P_i}^2} \log^2 \left( \frac{M_W^2}{M_{P_i}^2} \right)$$

- Controlling factor of the absolute scale for $m_\nu$ is $\nu_T$
- There is a G.I.M. cancellation bet $P_1^{\pm\pm}$ and $P_2^{\pm\pm}$
- It is further suppress by two helicity flips of internal charged lepton line
- Suppress by the 2-loop factor
- $(m_\nu)_{e^e}$ element is very small
- Expected to be in the sub-eV range
- If $Y_{ab}$ si not too weird $\rightarrow$ normal hierarchy
Active Neutrino mass matrix

\[
m_\nu = \tilde{f}(M_{P_1}, M_{P_2}) \times \begin{pmatrix}
m_e^2 Y_{ee} & m_e m_\mu Y_{e\mu} & m_e m_\tau Y_{e\tau} \\
m_e m_\mu Y_{e\mu} & m_\mu^2 Y_{\mu\mu} & m_\tau m_\mu Y_{\mu\tau} \\
m_e m_\tau Y_{e\tau} & m_\tau m_\mu Y_{\mu\tau} & m_\tau^2 Y_{\tau\tau}
\end{pmatrix}
\]

\[
= f(M_{P_1}, M_{P_2}) \times \begin{pmatrix}
2.6 \times 10^{-7} Y_{ee} & 5.4 \times 10^{-5} Y_{e\mu} & 9.1 \times 10^{-4} Y_{e\tau} \\
5.4 \times 10^{-5} Y_{e\mu} & 1.1 \times 10^{-2} Y_{\mu\mu} & 0.19 Y_{\mu\tau} \\
9.1 \times 10^{-4} Y_{e\tau} & 0.19 Y_{\mu\tau} & 3.17 Y_{\tau\tau}
\end{pmatrix},
\]

where

\[
\tilde{f}(M_{P_1}, M_{P_2}) = \frac{\sqrt{2} g^4 v_T \sin(2\delta)}{128\pi^4} \left[ \frac{1}{M_{P_1}^2} \log^2 \left( \frac{M_W}{M_{P_1}} \right) - \frac{1}{M_{P_2}^2} \log^2 \left( \frac{M_W}{M_{P_2}} \right) \right],
\]

and \( f = \tilde{f} \times (1\text{GeV}^2) \) gives a qualitatively estimate of the overall scale of active neutrino masses.
Neutrino Mass contd

- It is the NH

\[
\begin{pmatrix}
\varepsilon' & \varepsilon & \varepsilon \\
\varepsilon & 1 + \eta & 1 + \eta \\
\varepsilon & 1 + \eta & 1 + \eta
\end{pmatrix},
\]

where \( \varepsilon, \varepsilon' \) and \( \eta \ll 1 \).

- Relate to neutrino oscillation data via

\[
m^2_\nu = V^T_{PMNS} \begin{pmatrix}
m^2_1 & 0 & 0 \\
0 & m^2_2 & 0 \\
0 & 0 & m^2_3
\end{pmatrix} U V_{PMNS}, \quad U = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i\varphi_1} & 0 \\
0 & 0 & e^{i(\varphi_2 + \alpha)}
\end{pmatrix},
\]

where \( V_{PMNS} \) is the usual neutrino mixing matrix.
Scale of neutrino masses

and
Constraints from neutrino data

- The data

\[
7.1 \times 10^{-5} < \Delta m_{\odot}^2 < 8.9 \times 10^{-5} \text{ (eV}^2) , \quad 0.164 < \sin^2 \theta_{12} < 0.494 ,
\]
\[
1.4 \times 10^{-3} < |\Delta m_{\text{atm}}^2| < 3.3 \times 10^{-3} \text{ (eV}^2) , \quad 0.22 < \sin^2 \theta_{23} < 0.85 ,
\]
\[
\sin^2 2\theta_{13} = 0 \pm 0.04 . \tag{1}
\]

- We get six constraints from \( m_\nu^2 \)

- The first row

\[
f^2 Y_{e\tau}^2 \leq 1.32 \times 10^2 \text{ eV}^2 , \quad f^2 Y_{e\tau} Y_{\mu\tau} \leq 1 \text{ eV}^2 , \quad f^2 Y_{e\tau} Y_{\tau\tau} \leq 9.0 \times 10^{-2} \text{ eV}^2 .
\]

- Remaining three

\[
f^2 (Y_{\mu\mu}^2 + 300 Y_{\mu\tau}^2) \leq 2.7 \text{ eV}^2 ,
\]
\[
f^2 (Y_{\mu\mu} + 285 Y_{\tau\tau}) Y_{\mu\tau} \leq 2.4 \times 10^{-1} \text{ eV}^2 ,
\]
\[
f^2 (Y_{\mu\tau}^2 + 278 Y_{\tau\tau}^2) \leq 2.9 \times 10^{-2} \text{ eV}^2 .
\]
for $f = 0.5$ eV
Constraints from lepton rare decays

The tree level exchanges are the most stringent

- muonium anti-muonium conversion. The effective Hamiltonian from both $P_{1,2}^{\pm\pm}$ exchange

$$H_{M\bar{M}} = \frac{Y_{ee}Y_{\mu\mu}}{2M_{\bar{M}}^2} \bar{\mu} \gamma^\mu e_R \bar{\mu} \gamma_\mu e_R + h.c.,$$

Always in terms of reduced mass

$$\frac{1}{M_{\bar{M}}^2} = \frac{\sin^2 \delta}{M_{P_1}^2} + \frac{\cos^2 \delta}{M_{P_2}^2}$$

Current limit

$$Y_{ee}Y_{\mu\mu} < 2.0 \times 10^{-3} \left(\frac{M_{\bar{M}}}{100 \text{ GeV}}\right)^2.$$
Rare decays II

• Effective $e^+e^- \rightarrow l^+l^-, l = e, \mu, \tau$, contact interactions; The bounds are

\[
Y_{ee}^2 < 1.8 \times 10^{-3} (M_{--}/100 \text{ GeV})^2,
\]

\[
Y_{e\mu}^2 < 2.4 \times 10^{-3} (M_{--}/100 \text{ GeV})^2,
\]

\[
Y_{e\tau}^2 < 2.4 \times 10^{-3} (M_{--}/100 \text{ GeV})^2.
\]

• $\mu \rightarrow 3e$ and $\tau$ cousins. The BR’s give

\[
Y_{e\mu} Y_{ee} < 6.6 \times 10^{-7} (M_{--}/100 \text{ GeV})^2,
\]

\[
Y_{e\tau} Y_{ee} < 3.0 \times 10^{-4} (M_{--}/100 \text{ GeV})^2,
\]

\[
Y_{e\tau} Y_{\mu\mu} < 3.0 \times 10^{-4} (M_{--}/100 \text{ GeV})^2,
\]

\[
Y_{\mu\tau} Y_{\mu\mu} < 2.9 \times 10^{-4} (M_{--}/100 \text{ GeV})^2,
\]

\[
Y_{\mu\tau} Y_{ee} < 2.9 \times 10^{-4} (M_{--}/100 \text{ GeV})^2.
\]
**Rare decays III**

- Radiative decays $\mu \to e\gamma$, $\tau \to \mu(e)\gamma$ are less restrictive. The limits are

\[
Y_{e\mu}Y_{ee} < 6.6 \times 10^{-7} \left(\frac{M_{ee}}{100 \text{ GeV}}\right)^2,
\]

\[
Y_{e\tau}Y_{ee} < 3.0 \times 10^{-4} \left(\frac{M_{ee}}{100 \text{ GeV}}\right)^2,
\]

\[
Y_{e\tau}Y_{\mu\mu} < 3.0 \times 10^{-4} \left(\frac{M_{ee}}{100 \text{ GeV}}\right)^2,
\]

\[
Y_{\mu\tau}Y_{\mu\mu} < 2.9 \times 10^{-4} \left(\frac{M_{ee}}{100 \text{ GeV}}\right)^2,
\]

\[
Y_{\mu\tau}Y_{ee} < 2.9 \times 10^{-4} \left(\frac{M_{ee}}{100 \text{ GeV}}\right)^2.
\]

- Example with $f = 0.5$ eV and $M_{ee} = 400$ GeV. Lines from C.T and curve is $\tau \to 3e$
There are two amplitudes for this process

- The amplitude due to Majorana neutrino mass

\[ A_{\nu} \sim \frac{g^4}{M_W^4} \frac{m_{ee}}{<p>^2} , \]

- From doubly charged Higgs exchange

\[ A_{P_{1,2}} \sim \frac{g^4 Y_{ee} v_T \sin 2\delta}{16\sqrt{2}M_W^4} \left( \frac{1}{M_{P_1}^2} - \frac{1}{M_{P_2}^2} \right) , \]

- \[ A_{\nu}/A_{P_{1,2}} \lesssim 10^{-7} \]
Implication for LHC

• If $0\nu\beta\beta$ is seen it may **Not** be due to Majorana neutrino mass.
  
  $m_{ee}$ is vanishingly small

• In our model its due to doubly charged Higgs. Observation give values of $Y_{ee}$ and $M_{--}$

\[
Y_{ee} \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4
\]

\[
M_{Pi} \quad (GeV) \quad 300 \quad 400 \quad 500 \quad 600 \quad 700
\]

• LHC must find at least one doubly charged Higgs or rule out the model
Production of $P^{\pm\pm}$ at the LHC

- Single production via WW fusion same diagram as $0\nu\beta\beta$
- Pair production via Drell-Yan

\begin{align*}
  q & \to \gamma, Z^0 \to P_1^{\pm\pm} \\
  \bar{q} & \to \gamma, Z^0 \to P_1^{\mp\mp}
\end{align*}

- DY production is very robust. Must occur for all models with DCH.
- Model dependence in $Z$ exchange. In our case only on mixing angle of the two DCH.
**Production Cross section**

- **DY dominates over WW fusion** although kinematically not as favorable
- WW fusion is suppress because amplitude:
  1. $WWP^{\pm \pm}$ coupling has $\frac{\nu_h}{\nu}$
  2. Higher order in gauge coupling $g^4$
- **DY cross section** is large enough that LHC can produce them.
Signatures at the LHC

DHC has spectacular decays if Yukawa couplings are large enough

- Two body decays

\begin{align*}
(1) \quad P_{1}^{\pm\pm} & \rightarrow l_{aR}^{\pm} l_{bR}^{\pm} \quad (a, b = e, \mu, \tau), \\
(2) \quad P_{1}^{\pm\pm} & \rightarrow W^{\pm} W^{\pm}, \\
(3) \quad P_{1}^{\pm\pm} & \rightarrow P^{\pm} W^{\pm}, \\
(4) \quad P_{1}^{\pm\pm} & \rightarrow P^{\pm} P^{\pm}, \\
\end{align*}

The last one is energetically forbidden is our model

- The same sign leptonic modes can be flavor violating $e\tau, \tau\mu, \ldots$ (unmistakable)

- Helicity is always right-handed

- Doable for $\tau$?
Only 3-body decay allowed is

\[ P_1^{\pm\pm} \rightarrow W^\pm W^\pm X^0, \quad X^0 = T^0, h^0, P^0 \]

The couplings are

\[ P_1^{\pm\pm} l^\mp_{aR} l^\mp_{bR} : Y_{ab} s_\delta P_1^- l^e_{aR} l^e_{bR} + h.c. \]

\[ P_1^{\pm\pm} W^\mp_\mu P^\mp : ig c_\delta W^- [\partial_\nu P_1^{++} P^- - P_1^{++} \partial_\nu P^-] + h.c. \]

\[ P_1^{\pm\pm} W^\mp_\mu W^\mp_\nu X^0 : \frac{g^2}{\sqrt{2}} c_\delta c X P_1^{\pm\pm} W^\mp_\mu W^\mp_\nu X^0 + h.c. \]
Different Widths

- Long dash line $W^\pm W^\pm$
- Short dash line $W^\pm P^\pm$
- Solid lines for leptonic modes different Yukawa couplings
Conclusions

- Small Neutrino masses can be generated as a 2-loop process using DCH
- Agrees with all data.
- It connects origin of $m_\nu$ with physics that can be probed at LHC
- Production of DCH should be searched in its own right.
- $0\nu\beta\beta$ decay if observed does not necessary mean that it is due to Majorana neutrino masses.
- It does imply Majorana neutrinos masses at some level.
- LHC and $0\nu\beta\beta$ are strongly connected in this model