Nuclear Schiff Moments and Atomic EDMs

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T Violation and Atomic EDMs

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Handwaving Proof

More details.

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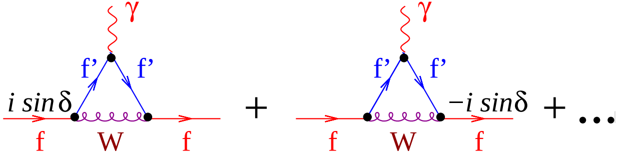
$$\boxed{\text{Of course, } \cancel{T} = \cancel{CP}.}$$

EDMs Sensitive to New Physics

In standard model only one phase. Diagrams cancel to high order, e.g.:



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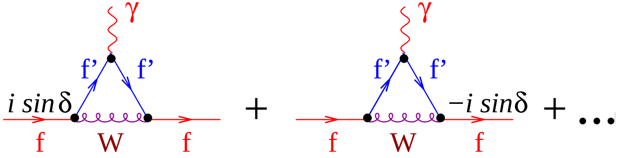
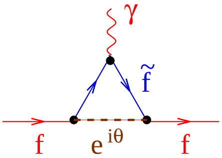


SUSY has many phases. Low-order diagrams uncanceled, e.g.:



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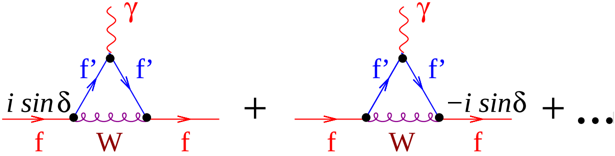

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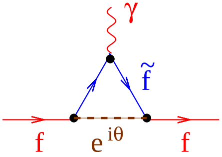

Thus, EDMs are insensitive to standard-model $\not{CP}$ but **sensitive to extra-standard-model $\not{CP}$**. Limits from atoms and neutrons, have already made SUSY a difficult proposition.

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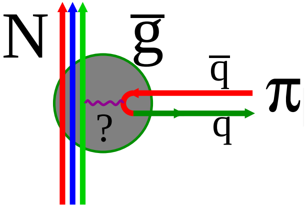
- Underlying fundamental theory generates three T-violating πNN vertices:



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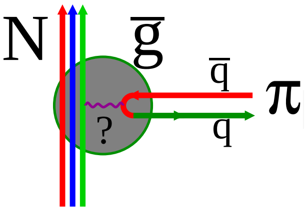
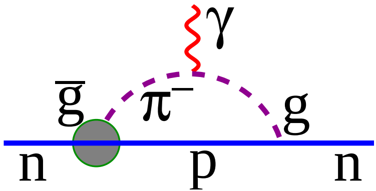
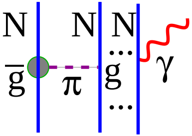


- Then neutron gets EDM from diagrams like this:



How Do Atoms Get EDMs?

- Nucleus can get one from nucleon EDM or **T-violating N*N* interaction:**



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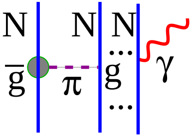

$$W \propto \left\{ \left[\bar{g}_0 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - \frac{\bar{g}_1}{2} \left(\tau_1^z + \tau_1^z \right) + \bar{g}_2 \left(3 \tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right) \right] (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) - \frac{\bar{g}_1}{2} \left(\tau_1^z - \tau_2^z \right) (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \right\} \cdot (\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2) \frac{\exp\left(-m_\pi |\mathbf{r}_1 - \mathbf{r}_2|\right)}{m_\pi |\mathbf{r}_1 - \mathbf{r}_2|}$$

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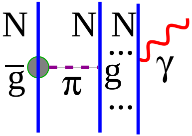

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- Finally, atom gets one from nucleus. Electronic *shielding* makes the relevant nuclear object the "Schiff moment"

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rather than dipole moment $\langle \vec{D} \rangle \equiv \langle \sum_p \vec{r}_p \rangle$.

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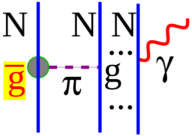

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Nuclear-structure theory's place in the chain: calculating dependence of $\langle \vec{S} \rangle$ on the $\bar{g}$'s in heavy nuclei.

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Hartree-Fock-Bogoluiobov calcualations with phenomenological density-dependent Skyrme interaction

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b_i, x_i, α adjusted to fit masses, radii, etc.

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Corrections to HFB are the frontier.

EDM Work Thus Far

In normal (non-octupole-deformed) nuclei, e.g., ^{199}Hg, the best work has been approximations to HFB with

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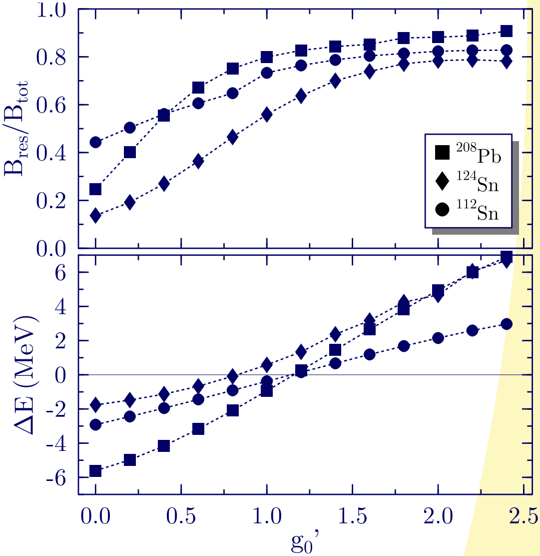
In octupole-deformed-nuclei, where Schiff moments are enhanced, treating W as an explicit perturbation is easier.

Constructing Good Skyrme Interaction

W probes spin density. Interaction should have good spin response. M. Bender et al. fit some time-odd terms of SkO′ to Gamow-Teller resonance energies and strengths.



Testing SkO′ and other Skyrme interactions

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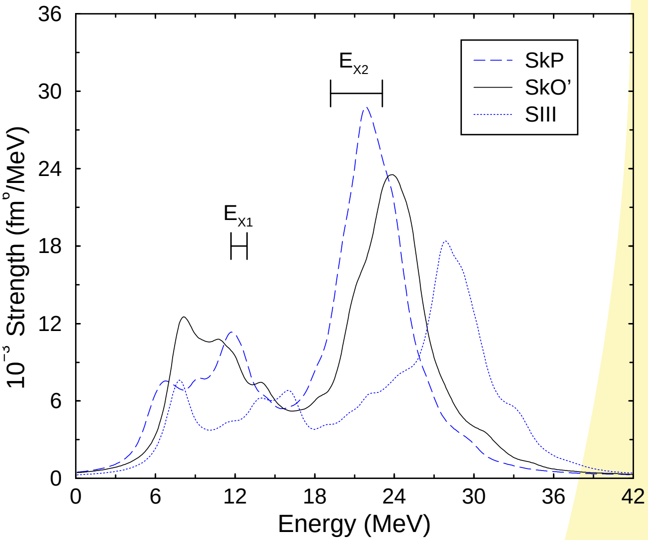

Results in ^{199}Hg

This is the nucleus with the best experimental limit.

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Is the spread a measure of uncertainty? Hard to know without intense focus on Skyrme functionals and related obervables.

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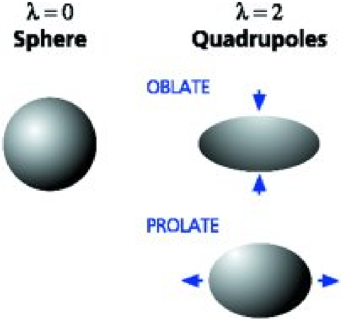

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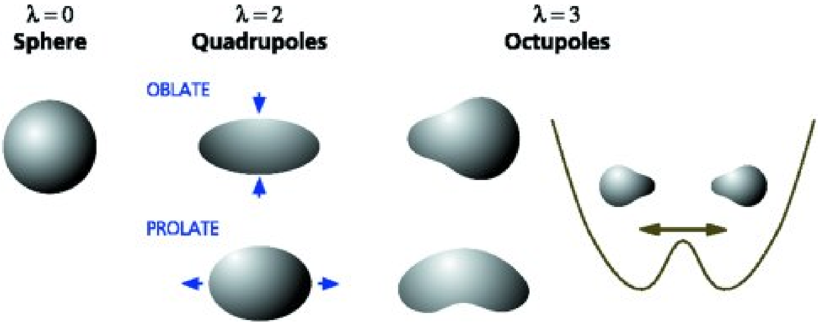
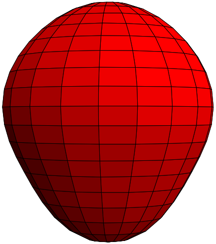


Schiff Moment with Octupole Deformation

Here we need to treat W as explicit perturbation:

$$\langle \vec{S} \rangle = \sum_m \frac{\langle 0|\vec{S}|m\rangle\langle m|W|0\rangle}{E_0 - E_m} + c.c.$$

where $|0\rangle$ is unperturbed ground state.



Calculated ^{225}Ra density

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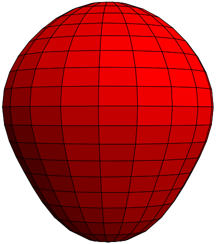

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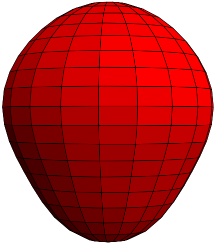

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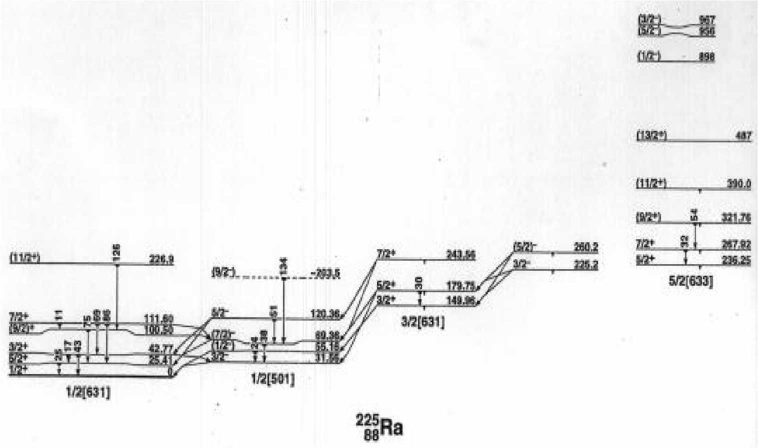
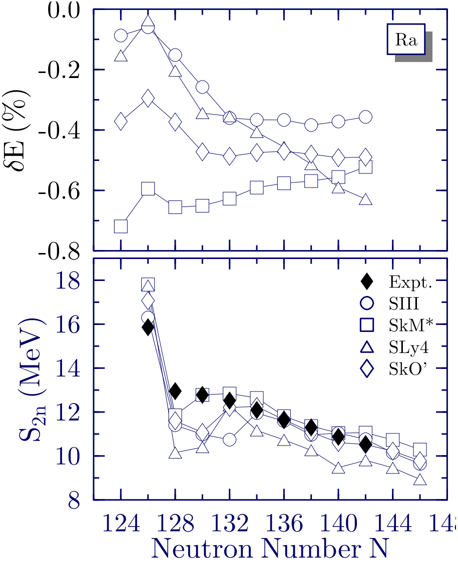
$\langle \vec{S} \rangle$ is large because $\langle \vec{S} \rangle_{\text{intr.}}$ is collective and $E_0 - E_{\bar{0}}$ is small.

Spectrum of ^{225}Ra



Testing Skyrme Interactions Again

Binding and Separation Energies



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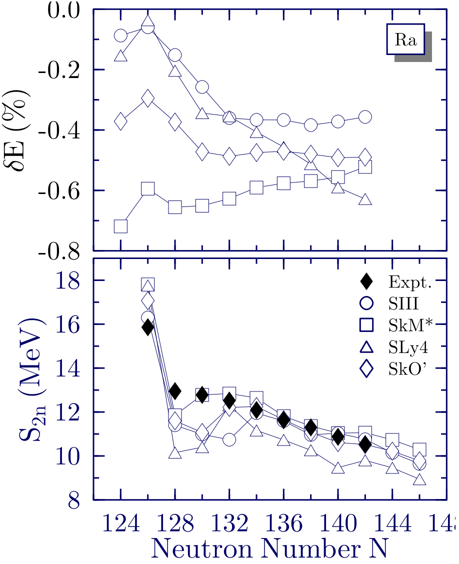

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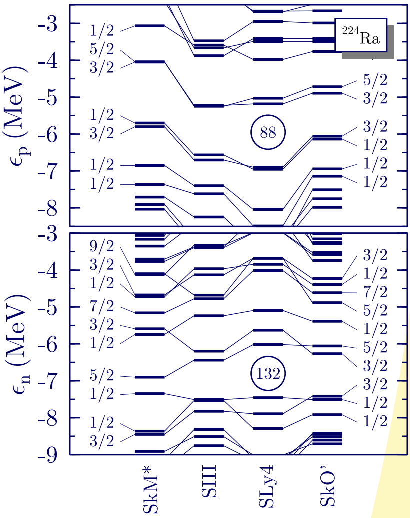
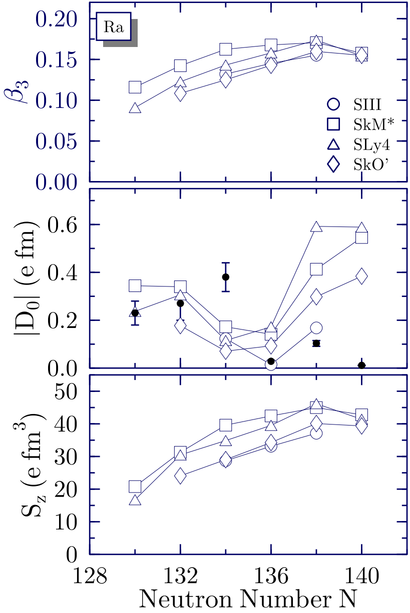


More Interaction Testing…

Octupole, Dipole, Schiff Stuff



^{225}Ra Results

Hartree-Fock calculation (Dobaczweski et al.) with SkO′ gives

$$\langle S_z \rangle_{\mathrm{Ra}} = -1.5\, g\bar{g}_0 \; + 6.0\, g\bar{g}_1 \; - 4.0\, g\bar{g}_2 \;\; (\mathrm{e\ fm}^3)$$

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What can we do to reduce it?

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- Skyrme interactions are limited.

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- More exact Schiff operator
- Parity- and angular-momentum projection
- particle-hole correlations

But, *as in $\beta\beta$ decay*, uncertainties may not shrink much.
Spin-dependent two-body operators for which no data exist pose problems because

- The operators are two-body and spin-dependent
- There are no data
- Skyrme interactions are limited.
- Need more *related* data: isoscalar-dipole distributions, spin-multipole distributions,. . . (like in $\beta\beta$)

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THE END

What Do EDMs Have to Do With T

Consider nondegenerate ground state $|\mathrm{g.s.} : J, M\rangle$. Symmetry under rotations $R_y(\pi)$ for vector operator like $\boxed{\vec{d} \equiv \sum_i e_i \vec{r}_i}$,

$$\langle \mathrm{g.s.} : J, M | \; \vec{d} \; | \mathrm{g.s.} : J, M\rangle = -\langle \mathrm{g.s.} : J, -M | \vec{d} | \mathrm{g.s.} : J, -M\rangle \; .$$

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T takes M to $-M$, like $R_y(\pi)$. But $\vec{d}$ is *odd* under $R_y(\pi)$ and *even* under T, so for T conserved

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If T is violated, argument fails because T can take $|\text{g.s.} : JM\rangle$ to $|\text{ex.} : J, -M\rangle$, a state in a *different* multiplet.

Return

Shielding by Electrons

Unfortunately for atomic experiments:

Theorem (Schiff)

The nuclear dipole moment causes the atomic electrons to rearrange themselves so that they develop a dipole moment opposite that of the nucleus. In the limit of nonrelativistic electrons and a point nucleus the electrons' dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes!

Proof

Consider atom with nonrelativistic constituents (with dipole moments $\vec{d}_k$) held together by electrostatic forces. The atom has a "bare" edm $\boxed{\vec{d} \equiv \sum_k \vec{d}_k}$ and a Hamiltonian

$$H = \sum_k \frac{p_k^2}{2m_k} + \sum_k V(\vec{r}_k) - \sum_k \vec{d}_k \cdot \vec{E}_k$$

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K.E. + Coulomb

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= & \qquad H_0 & + i \sum_k (1/e_k) \left[\vec{d}_k \cdot \vec{p}_k , H_0 \right]
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The perturbing Hamiltonian

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So the net EDM is zero!